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TEST OF THE ELLIS-JAFFE SUM RULE USING PARAMETRIZATION OF THE MEASURED LEPTON-PROTON ASYMMETRY

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It is shown that the values of the first moment of $g_1^p(x)$ calculated from simultaneous fits of the SMC and E143 data on the asymmetries $A_1^p(x)$ are below the Ellis-Jaffe prediction by more than 7σ.

The investigation has been performed at the Laboratory of Particle Physics, JINR.

Проверка правила сумм Эллиса-Джаффе с использованием параметризации данных по лептон-протонной асимметрии

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Показано, что величины первого момента от $g_1^p(x)$, вычисленные из одновременной аппроксимации SMC и E 143 данных по асимметрии $A_1^p(x)$, находятся ниже предсказаний правила сумм Эллиса-Джаффе более чем на 7σ.

Работа выполнена в Лаборатории сверхвысоких энергий ОИЯИ.

1.Introduction

It has been found from the measurements of the spin-dependent structure function $g_1^p(x)$ of the proton by the SMC [1] and E143 [2] that the value Γ_1^p, which is the first moment of $g_1^p(x)$, is below the Ellis-Jaffe sum rule prediction. We quote in detaial the results of Refs. [1], [2], because they are important for our discussion.

1.1. *SMC*. The result for the first moment of $g_1^p(x)$ at $\langle Q^2\rangle = 10$ GeV^2 is

$$\Gamma_1^p = \int_0^1 g_1^p(x)dx = 0.136 \pm 0.011 \text{ (stat.)} \pm 0.011 \text{ (syst.)}. \qquad (1)$$

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The integral over the measured x range is

$$\int_{0.003}^{0.7} g_1^p(x)dx = 0.131 \pm 0.011 \pm 0.011. \quad (2)$$

The values of integrals over unmeasured x regions are

$$\int_{0}^{0.003} g_1^p(x)dx = 0.004 \pm 0.002, \quad \int_{0.7}^{1} g_1^p(x)dx = 0.001 \pm 0.001. \quad (3)$$

The corresponding Ellis-Jaffe prediction corrected for QCD effects [3] is:

$$\Gamma_1^p = 0.176 \pm 0.006. \quad (4)$$

SMC has evaluated $g_1^p(x)$ from virtual photon-proton asymmetry $A_1^p(x, Q^2)$ averaged over Q^2 in each bin using the relation:

$$g_1^p(x) \cong \frac{A_1^p(x, Q^2)\, F_2^p(x, Q^2)}{2x\,(1 + R(x, Q^2))} \cong A_1^p(x) F_1^p(x, Q^2). \quad (5)$$

$A_1^p(x,Q^2)$ is assumed to be independent of Q^2. The unpolarized structure functions $F_2^p(x, Q^2)$ and $R(x, Q^2)$ were taken from parametrizations [4] and [5], respectively, for the average $\langle Q^2 \rangle = 10$ GeV^2 in the SMC kinematic region. The virtual photon-proton asymmetry A_1^p is related to the measured muon-proton asymmetry A^p:

$$A^p = \frac{\sigma^{\uparrow\downarrow} - \sigma^{\uparrow\uparrow}}{\sigma^{\uparrow\downarrow} + \sigma^{\uparrow\uparrow}}, \quad (6)$$

$$A_1^p = \frac{A^2}{D} - \eta A_2^p, \quad (7)$$

where $\sigma^{\uparrow\downarrow}(\sigma^{\uparrow\uparrow})$ is the cross section for the longitudinally polarized muons scattering on protons polarized opposite (along) to the muon momentum, D and η are the kinematic factors. SMC has shown in the separate experiment [6] that asymmetry A_2^p arising from the interference between virtual photons with transverse and longitudinal polarizations is compatible with zero within statistical errors. In addition, since coefficient η in the SMC kinematic region is small, product ηA_2^p can be neglected in Eq.(7). So, with this assumption asymmetry A_1^p is directly proportional to the measured A^p:

$$A_1^p \cong \frac{A^p}{D}. \tag{8}$$

1.2. *E143*. The result for the first moment of $g_1^p(x)$ at $\langle Q^2\rangle = 3$ is

$$\Gamma_1^p = 0.127 \pm 0.004(\text{stat.}) \pm 0.010(\text{syst.}). \tag{9}$$

The integral over the measured range is

$$\int_{0.029}^{0.8} g_1^p(x)\,dx = 0.120 \pm 0.004 \pm 0.008. \tag{10}$$

The values of integrals over unmeasured x regions are

$$\int_{0}^{0.029} g_1^p(x)\,dx = 0.006 \pm 0.006, \quad \int_{0.8}^{1} g_1^p(x)\,dx = 0.001 \pm 0.001. \tag{11}$$

The corresponding Ellis-Jaffe prediction corrected for QCD effects [3] is:

$$\Gamma_1^p = 0.160 \pm 0.006. \tag{12}$$

E143 has evaluated g_1^p from the measurements of $A_{\|}$ and $A_{\perp}$ asymmetries for the scattering of the longitudinally polarized electrons on the target polarized parallel and transverse to the beam direction:

$$\frac{g_1^p}{F_1^p} = D^{-1}(A_{\|} + \tan(\Theta/2)\,A_{\perp}), \tag{13}$$

where Θ is the electron scattering angle. This ratio is related to the virtual photon-proton asymmetry A_1^p:

$$A_1^p = \frac{g_1^p}{F_1^p} - \gamma^2 \frac{g_2^p}{F_1^p}, \quad \gamma^2 = \frac{Q^2}{\nu^2} \ll 1 \tag{14}$$

and with the same level of confidence as for SMC, we can neglect product $\gamma^2 g_2^p/F_1^p$ and obtain the same approximate relation as Eq.(5):

$$A_1^p \cong \frac{g_1^p}{F_1^p}. \tag{15}$$

Calculating Γ_1^p from g_1^p/F_1^p, in E143 it was assumed that the last ratio is independent of Q^2. Such an assumption was justified by observing good agreement between SLAC and SMC data taken at different Q^2.

The two experiments obtained Γ_1^p, which is below the Ellis-Jaffe sum rule prediction by more than two standard deviations. These discrepancies could be caused by some physics effects not taken into account by the Ellis-Jaffe model or due to imperfection of the data and their analysis. Since in terms of standard deviations the difference between the data and theory is not significant, it is worth-while to examine possible experimental problems. One of them is a «nonsmoothness» in x behaviour of A_1^p present in both experiments and which is difficult to explain by statistical errors. It might be produced by possible incorrectable point-to-point systematic fluctuations. On the other hand, both experiments did not take into account the theoretical prediction $A_1^p(x=1)=1$ while extrapolating the data to the unmeasured region $0.7<x\leq 1$. These two observations motivated our study of the changes in the Γ_1^p when the latter is calculated with the constraint $A_1^p(1)=1$ and with smooth input $A_1^p(x)$. Such an approach has been used in paper [10] for discussions of the Bjorken sum rule tests.

2. Calculation of Γ_1^p

We calculate the first moment of g_1^p as follows:

1) The x-dependence of A_1^p is parametrized by function $A^f(x)$ (the form of the function will be discussed later) with some free parameters. As long as values $A_1^p(x, Q^2)$ and g_1^p/F_1^p measured in SMC and E143 are independent of Q^2, we can fit the data from two experiments simultaneously.

2) This function A^f is used for calculations of $g_1(x)$ and its integral:

$$g_1^p(x) \cong A^f(x, P_1, P_2)\,\frac{F_2^p(x, Q^2)}{2x(1+R(x, Q^2))} \cong A^f(x, P_1, P_2)F_1^p(x, Q^2), \tag{16}$$

where we have used for $F_2^p(x, Q^2)$ the NMC parametrization [4] and the SLAC parametrization [5] for $R(x, Q^2)$ at given Q^2. Then

$$\Gamma_1^p = \int_0^1 g_1^p\,dx = \int_0^1 A^f(x, P_1, P_2,..)F_1^p(x)\,dx, \tag{17}$$

where $P_1, P_2, \ldots$ are parameters obtained from fit of the measured $A_1^p(x)$. The error of the integral from Eq.(17) is calculated using errors of the parameters taken from the fits (see Section 4).

3. Parametrization of A_1^p

3.1. *The Choice of Functions for Fitting.* We have suggested that the form of the parametrization functions should be the simplest one with the minimal number of parameters. These functions must meet two requirements: $A_1^p(0) = 0$ and $A_1^p(1) = 1$ taken from theoretical predictions [7, 8]. Two functions have been chosen out of many:

$$A_1^f(x) = \frac{P_2}{2} \cdot (x + x^{P_1}), \tag{18}$$

$$A_2^f(x) = P_2 \cdot x^{P_1}, \tag{19}$$

where P_1, P_2 are free parameters.

3.2. *The Test of Agreement between SMC and E143 Data.* To test the consistency of the SMC data on A_1^p and the E143 data on g_1^p/F_1^p, we have performed fits with functions from

Table 1. Separate fits of the SMC and E143 data using functions from Eqs.(18), (19)

The form of function	Experiment	P_1	ΔP_1	P_2	ΔP_2	$\chi^2/d.o.f.$
$A_1^f(x)$	SMC	0.551	0.084	0.988	0.158	0.58
	E143	0.625	0.038	1.100	0.054	1.43
$A_2^f(x)$	SMC	0.665	0.082	0.888	0.172	0.58
	E143	0.747	0.032	1.043	0.060	1.38

Table 2. Test of the systematic shift between the SMC and E143 data

The value of P_2	Function	$\chi^2/d.o.f.$	P_1	ΔP_1	P_2	ΔP_2
a) P_2 is free for E143 and $P_2 = 1$ for SMC	$A_1^f(x)$	1.167	0.599	0.031	1.068	0.046
	$A_2^f(x)$	1.123	0.732	0.025	1.016	0.043
b) P_2 is free fro SMC and $P_2 = 1$ for E143	$A_1^f(x)$	1.227	0.560	0.016	1.004	0.087
	$A_2^f(x)$	1.126	0.725	0.012	1.005	0.087

Eqs.(18), (19) for each experiment separately taking into account only statistical errors. From the results of the fits which are shown in Table 1 we conclude that the data are consistent because the values of parameters P_1 and P_2 are the same within the errors.

Table 1 also shows that within the errors $P_2 \approx 1$ to be expected if the theoretical prediction $A_1^p(1) = 1$ is valid. So, we can use $P_2 = 1$ unless there is a systematic shift between the SMC and E143 data. It was checked by fitting the data simultaneously for two cases according to different assumptions on P_2:

1) $P_2 = 1$ for the SMC data and free for the E143 data;

2) P_2 is free for the SMC data and $P_2 = 1$ for the E143 data.

Table 2 shows that within the errors $P_2 \approx 1$ as expected if there is no systematic shift between the data, which justifies the use $P_2 = 1$ for further fits.

3.3. *The Results of Fitting*. The experimental points for fits were taken either with statistical errors only or with statistical and systematic errors combined.

Table 3 and Figures 1, 2 show the results of the fits of the SMC and E143 data taken either separately or simultaneously (SMC+E143) by Eqs.(18), (19) assuming that $P_2 = 1$.

Table 3. Separate and simultaneous fits of the data on A_1 taken with:
a) the statistical errors only;
b) statistical and systematic errors combined linearly;
c) statistical and systematic errors combined in quadratures

The form of function	SMC			E143			SMC+E143		
	P_1	ΔP_1	$\chi^2/d.o.f.$	P_1	ΔP_1	$\chi^2/d.o.f.$	P_1	ΔP_1	$\chi^2/d.o.f.$
a) $A_1^f(x)$	0.556	0.044	0.524	0.561	0.016	1.509	0.560	0.015	1.192
b) $A_1^f(x)$	0.562	0.067	0.252	0.569	0.025	0.673	0.568	0.024	0.537
c) $A_1^f(x)$	0.559	0.050	0.431	0.569	0.016	1.197	0.565	0.018	0.950
a) $A_2^f(x)$	0.712	0.036	0.560	0.726	0.012	1.344	0.725	0.011	1.092
b) $A_2^f(x)$	0,717	0.054	0.270	0.730	0.019	0.596	0.728	0.018	0.537
c) $A_2^f(x)$	0.715	0.041	0.460	0.728	0.014	1.059	0.727	0.013	0.866

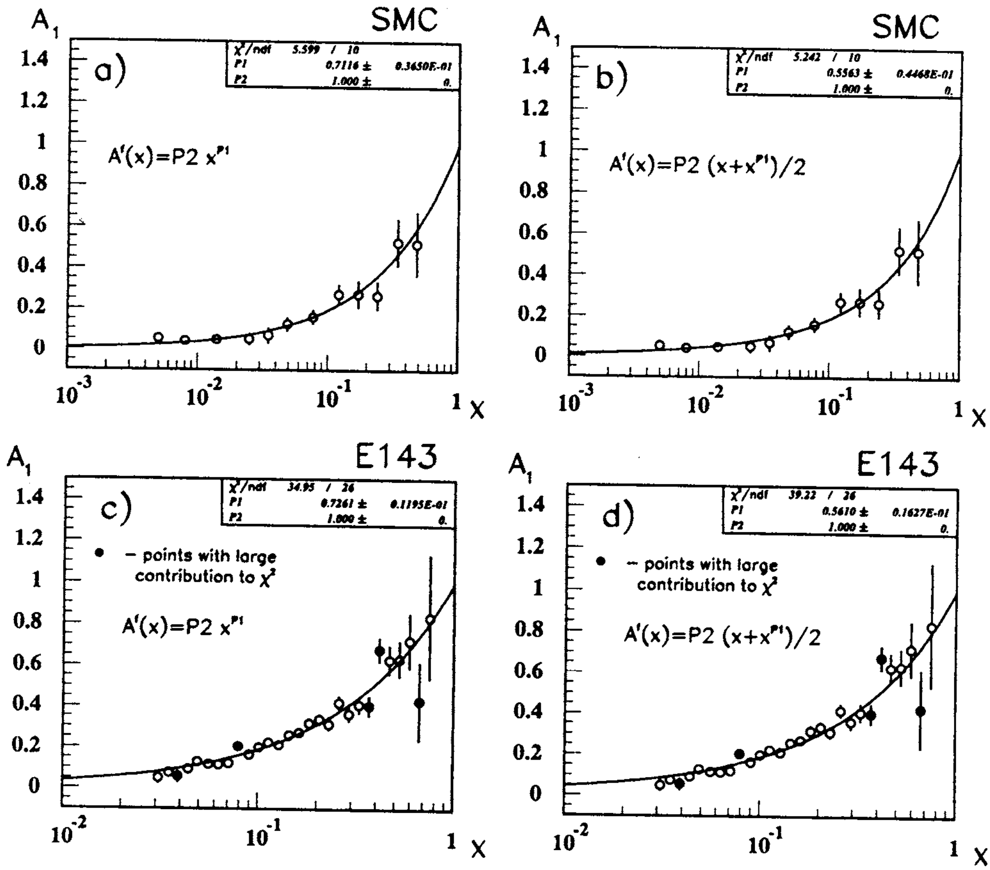

Fig.1. The approximations of lepton-proton asymmetry measured by the SMC and E143. The statistical errors are taken only

One can see that both fits yield consistent results for the free parameter P_1 when the same function is used. From χ^2 values we can conclude that the data from both experiments are very well parametrized using the simplest functions with one free parameter. We cannot make the choice between the two functions because values $\chi^2/d.o.f.$ are rather good in both cases. They are however worce for the E143 data indicating the presence of some systematic point-to-point fluctuations. For example, the points at $x=0.039$, 0.079, 0.370, 0.416 and 0.666 (marked as dark points in Figs. 1c and 1d) give respectively 3.2, 5.9, 3.6, 5.6 and 3.2 units to χ^2 of the total 34.95 for 26 degrees of freedom. These contributions are largely reduced if the systematic errors (compare $\chi^2/d.o.f.$ in Table 3) are taken into account.

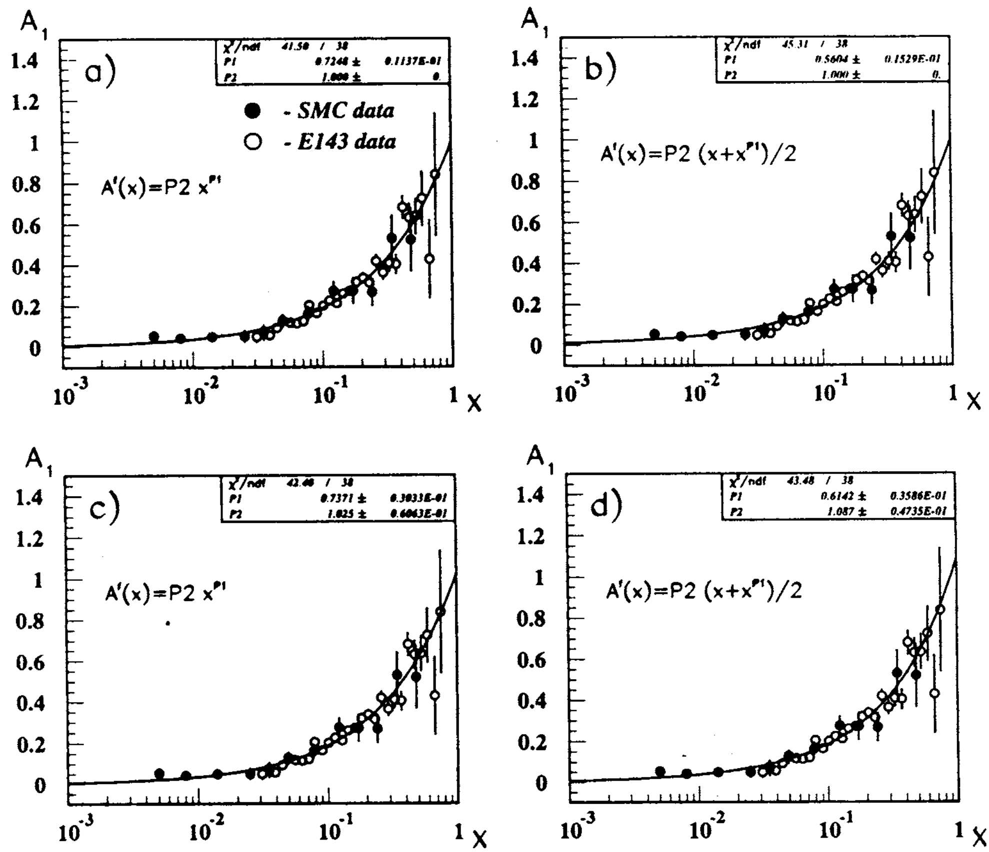

Fig.2. The approximations of proton asymmetry for the (SMC+E143) data. The statistical errors are taken only. The fits c) and d) were performed using functions with two free parameters

4. The First Moment of g_1^p

The functions with parameters of Table 3 were used to calculate integrals from Eq.(17). For the SMC and E143 data the structure functions F_2 and R have been taken at $Q^2 = 10$ and $3\,\mathrm{GeV}^2$, respectively. The integral Γ_1^p for (SMC+E143) data has been calculated using the parameters of the simultaneous fit (column (SMC+E143), Table 3) and structure functions F_2 and R at $Q^2 = 5\ \mathrm{GeV}^2$. Uncertainties of the integrals were calculated

Table 4. The values of integrals Γ_1^p for the SMC data on A_1 ($\langle Q^2\rangle = 10$ GeV ^{2}) taken with:
a) statistical errors only;
b)statistical and systematic errors combined in quadratures

The form of A^f function	Γ_1^p	$\int_{0.003}^{0.7} g_1^p(x)\,dx$	$\int_{0}^{0.003} g_1^p(x)\,dx$	$\int_{0.7}^{1} g_1^p(x)\,dx$
a) $A_1^f(x)$	0.142±0.013	0.130±0.010	0.010±0.003	0.0010±0.0004
b) $A_1^f(x)$	0.141±0.015	0.129±0.011	0.010±0.004	0.0020±0.0004
a) $A_2^f(x)$	0.139±0.012	0.131±0.010	0.006±0.002	0.0020±0.0004
b) $A_2^f(x)$	0.138±0.013	0.130±0.011	0.006±0.002	0.0020±0.0004

Table 5. The values of integrals Γ_1^p for the E143 data on A_1 ($\langle Q^2\rangle = 3$ GeV ^{2}) taken with:
a) statistical errors only;
b)statistical and systematic errors combined in quadratures

The form of A^f function	Γ_1^p	$\int_{0.029}^{0.8} g_1^p(x)\,dx$	$\int_{0}^{0.029} g_1^p(x)\,dx$	$\int_{0.8}^{1} g_1^p(x)\,dx$
a) $A_1^f(x)$	0.129±0.04	0.106±0.002	0.022±0.002	0.0010±0.0004
b) $A_1^f(x)$	0.127±0.004	0.105±0.002	0.021±0.002	0.0010±0.0004
a) $A_2^f(x)$	0.125±0.003	0.107±0.002	0.016±0.001	0.0020±0.0004
b) $A_2^f(x)$	0.124±0.003	0.107±0.002	0.016±0.001	0.0010±0.0004

Table 6. The values of integrals Γ_1^p for the SMC+E143 data on A_1 ($Q^2 = 5$ GeV ^{2}) taken with:
a) statistical errors only;
b)statistical and systematic errors combined in quadratures

The form of A^f function	Γ_1^p	$\int_{0.003}^{0.8} g_1^p(x)\,dx$	$\int_{0}^{0.003} g_1^p(x)\,dx$	$\int_{0.8}^{1} g_1^p(x)\,dx$
a) $A_1^f(x)$	0.133±0.004	0.124±0.003	0.008±0.001	0.0010±0.0004
b) $A_1^f(x)$	0.132±0.004	0.124±0.003	0.007±0.001	0.0010±0.0003
a) $A_2^f(x)$	0.128±0.003	0.123±0.003	0.040±0.003	0.0010±0.0003
b) $A_2^f(x)$	0.128±0.004	0.123±0.003	0.004±0.001	0.0010±0.0003

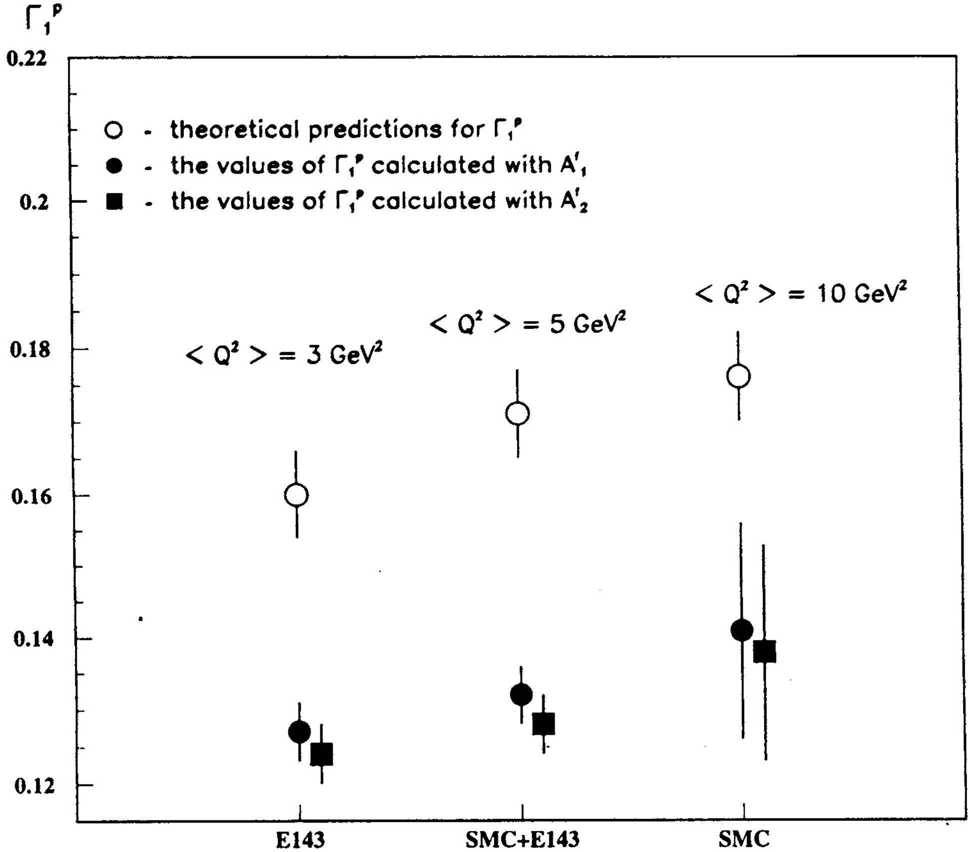

Fig.3. The comparison between theoretical predictions for Γ_1^p and values obtained with the proposed method and errors $\Delta\Gamma_1^p$ estimated from statistical and systematic errors of the data combined in quadratures

by shifting average value of the parameter P_1 by +/− one standard deviation: $(P_1 - \Delta P_1)$ and $(P_1 + \Delta P_1)$. The results of calculations are presented in Tables 4—6. As it is seen from Tables 4 and 5, the integrals over the entire region calculated by us coincide within the errors with those from publications [1, 2].

We have also calculated the integrals for the measured and unmeasured x range to compare our results with those from Refs. [1, 2] (see Eqs.(2), (3) and Eqs. (10), (11)). The integrals over the measured x range for the SMC data calculated by us and in Ref. [1] are the same, but extrapolation to $x=0$ is understimated in Ref. [1]. For the E143 data the integrals over the measured region are overstimated in Ref. [2] while the extrapolation to $x=0$ is largely underestimated. The results for the E143 measured region have smaller errors than those in paper [2] due to obvious reasons related to substitution of the experimental points by a smooth parametrization and additional constraints at $x=0$ and $x=1$. We have also computed Γ_1 from the SMC and E143 data at a common value of $Q_2 = 5$ GeV$_2$ by fitting simultaneously reevaluated values of $g_1(x)$. Reevaluation of $g_1(x)$ has been done in the same way as in [9]. The results are to be compared with the Ellis-Jaffe prediction for Γ_1 at 5 GeV$_2$, which is 0.171 ± 0.006 (see Figure 3). The comparison between theoretical predictions on Γ_1^p and the values obtained with the proposed method are also presented in Figure 3.

5. Discussion

1. It is shown that lepton-proton asymmetry from the SMC and E143 data fitted either separately or simultaneously can be parametrized using the simplest functions with one free parameter only. The SMC and E143 data are in agreement with the theoretical predictions $A_1^p(x=0)=0$, $A_1^p(x=1)=1$. These constrains can be used in data parametrization.

2. The method to calculate Γ_1^p from Eq.(10) using parametrization of asymmetry $A_1^p(x)$ is a natural generalization of the $g_1^p(x)$ calculations from Eq.(5), when parametrizations $F_2^p(x, Q^2)$ and $R(x, Q^2)$ are used instead of experimental values. The values of $\int g_1^p(x)\, dx$ calculated with this method for the entire x range are in agreement with the published ones:

	Γ_1^p published value	Γ_1^p this paper
SMS data	0.136 ± 0.016	0.141 ± 0.015
E143 data	0.127 ± 0.011	0.127 ± 0.004

where the statistical and systematic errors are combined in quadratures.

3. The use of the parametrization of the measured asymmetries with physics constraints at $x=0$ and $x=1$ can be helpful in revealing unaccounted systematic errors in the data. For

example, the comparison of $\int g_1^p(x)\,dx$ over the measured x range with that of Ref. [2] (E143) indicates that the last one is largely overestimated:

	E143[2]	This paper
$\int_{0.029}^{0.8} g_1^p(x)\,dx$	0.120 ± 0.009	0.105 ± 0.003 .

We explain larger value of the integral from Ref. [2] by pretty large fluctuations of some data pionts at intermediate x. Due to the same reason the unconstrained fit (free P_2) of the E143 data yields $A_1^p(x=1) > 1$ (see Figs. 2c, 2d).

4. The parametrization of the asymmetries with the constraints at the boundaries provides a law for the extrapolations to the unmeasured low x and high x regions. This law is suggested by the data themselves, which we consider as more justified than making assumptions similar to those of Refs. [1, 2]. We find, for example, that E143 underestimates the low x contribution to the integral:

	E143 paper	This paper
$\int_{0.0}^{0.029} g_1^p(x)\,dx$	0.006 ± 0.004	0.021 ± 0.003 .

This difference can serve as an argument to perform better measurements in the low x range for the proper choice of the parametrization.

5. The Γ_1^p calculated from the parametrized asymmetries with the constraints at the boundaries have smaller errors than those of Refs. [2, 9]. This indicates overestimation of possible systematic errors in these papers which devaluates the results of the measurements when compared to the Ellis-Jaffe predictions.

The proposed method allows one to demonstrate, that the conclusion of Ref. [2] that Γ_1^p is more than two standard deviations velow the Ellis-Jaffe sum rule predictions is dominated by systematic errors. The values of the first moment of g_1^p calculated by the proposed method from SMC and E143 data are also smaller than theoretical predictions, but the significance of deviation from them is now larger. For example, the integrals Γ_1^p calculated from the fits of the SMC, E143 and (SMC+E143) data on A_1^p (taken with statistical and systematic errors combuned in quadratures) are below the Ellis-Jaffe predictions by 2.5, 10 and 9σ, respectively. These results can be considered as a clear proof of the violation of the Ellis-Jaffe sum rule.

6. The value of Γ_1^p depends only slightly on the $A_{1,2}^f$ parametrization and the present accuracy of the data does not permit one to choose between them.

7. Concerning the shortcomings of the method it should be emphasized that the calculation of the errors for Γ_1^p can be improved using a more sophisticated procedure for the treatment of experimental errors and their correlations. But we believe that this procedure will not change substantially the above conclusions. Our belief is based on the comparison of the P_1 and ΔP_1 values given in Table 3. For the Table 3 (line b) results we have taken each experimental point with the error equal to the linear sum of the statistical and systematic errors, i.e., the upper limit of the possible error; using P_1 and ΔP_1 from these fits for the estimations of the Γ_1^p and $\Delta\Gamma_1^p$ we have found that difference between the SMC+E143 data and the Ellis-Jaffe prediction will be 7.5σ instead of 9σ obtained in case of more common treatment of errors.

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